Acoustic Mode Determination in Solid Rocket Motor Stability Analysis

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Nomenclature

a	= speed of sound
A	= dimensionless acoustic admittance, $= \rho_0 a_0 (u'/p')$
e_b	= web distance burned
E_N^2	$= E_N^2 = \int_v \tilde{p}_N^2 \mathrm{d}v$
f	= frequency
i	$= \sqrt{-1}$
Im( )	= imaginary part of a complex quantity
k	= wavenumber, $= (\omega + i\alpha)/a_0$
L	= length
M	= Mach number
n	= outward pointing unit normal vector
p	= pressure
R	= internal radius
Re( )	= real part of a complex quantity
S	= surface area enclosing the acoustic cavity
u	= velocity
V	= volume of the acoustic cavity
z	= axial coordinate
α	= damping coefficient
γ	= ratio of specific heats
ρ	= density
ω	= angular frequency $2\pi f$

Subscripts

b	= value taken at the burning surface
L	= value taken at the acoustic cavity aft closure plane
N	= reference acoustic mode
0	= reference steady state

Superscripts

$(\ )^*$	= value taken at acoustic boundary-layer edge
$(\ )'$	= unsteady harmonic component
$(\tilde{\ })$	= spatial component of a harmonic function
$(\bar{\ })$	= steady-state value

Introduction

THE determination of acoustic reference modes is almost a mandatory step in solid rocket motor stability analysis. An exception to this statement is provided by the recent development of computer codes that solve the full unsteady equations of motions inside the rocket motor, including the choked ejection nozzle. However, these codes presently do not yet represent the state-of-the-art and both the engineer and the research scientist need to rely on more conventional, less expensive methods to study the natural unsteady behavior of solid rocket motors. These conventional analyses—such as the acoustic balance method[1-3]—call for the determination of reference acoustic modes of the rocket engine combustion chamber. These modes are used primarily to evaluate the stability integrals that define the driving or damping contributions to the chamber stability; consequently, the accuracy of the stability analyses is directly governed by the quality of the acoustic mode determination. The critical steps in evaluating such acoustic modes are 1) the definition of a closed cavity in which to perform the acoustic calculations and 2) the choice of acoustic boundary conditions. The cavity is usually taken to be limited by the head end of the combustion chamber, the burning surface, inert walls in certain cases, and an aft closure plane that formally separates the low-velocity flowfield inside the chamber from the high-velocity flowfield inside the nozzle (Fig. 1). It is of primary importance that the aft closure plane be located within the low Mach number domain because stability analyses assume a quasi-incompressible mean flowfield.[1-3] Apart from this restriction, the precise location of this aft closure plane is theoretically free. Obviously, an ideal acoustic mode determination should ensure that the stability integrals are insensitive to the position of the aft closure plane within the incompressible domain.

It is the purpose of this work to show that the classical rigid-wall assumption used to evaluate the chamber acoustic modes does not ensure the independence of the stability integrals from the location of the aft closure plane. This point is particularly crucial when the nozzle length is not small compared to the motor length. A simple acoustic admittance boundary condition will be proposed that greatly improves the stability analyses through an improved reference acoustic mode determination.

Test Case

The test case considered is that of a small rocket motor which is part of an experimental apparatus used at the ONERA Palaiseau Research Center to evaluate pressure-coupled response functions of solid propellants at high frequencies.

The apparatus is described in detail in Refs. 4 and 5. Figure 2 shows the motor geometry. For the present work, one-dimensional axial acoustic stability predictions are performed for different acoustic mode determinations. The parameters defining the motor operating conditions are assumed to be held fixed at the following values: $p_0 = 3$ MPa, $a_0 = 1056$ m/s, $\gamma = 1.213$, and $\rho_0 = \gamma p_0 / a_0^2 = 3.263$ kg/m^3. Moreover, a zero acoustic burning surface admittance, $A_b = 0$, is assumed for all subsequent acoustic stability calculations. This latter assumption is not restrictive and has been made only for the sake of comparison between different acoustic mode determinations.

Classical Stability Analysis

Application of the acoustic balance theory[2,3] leads to the following formula for the complex wavenumber:

$$k^2 = k_N^2 + \frac{ik_N}{E_N^2} \int_s \tilde{p}_N \left(\rho_0 a_0 \frac{u'^*}{p'} + \frac{\bar{u}}{a_0} \right) \cdot n \mathrm{d}s \qquad (1)$$

No flow turning terms appear in Eq. (1) as they have been shown to be inaccurate[5]; instead, the admittance correction approach[5,6] has been preferred. p_N is the solution of the following unperturbed one-dimensional wave equation:

$$\frac{\mathrm{d}^2 \tilde{p}_N}{\mathrm{d}z^2} + \frac{1}{R^2} \frac{\mathrm{d}R^2}{\mathrm{d}z} \frac{\mathrm{d}\tilde{p}_N}{\mathrm{d}z} + k_N^2 \tilde{p}_N = 0 \qquad (2)$$

with the following boundary conditions:

$$z = 0 \quad \tilde{p}_N = \tilde{P}_{N0} \quad \text{and} \quad \frac{\mathrm{d}\tilde{p}_N}{\mathrm{d}z} = 0$$

$$z = L \quad \frac{\mathrm{d}\tilde{P}_N}{\mathrm{d}z} = 0 \qquad (3)$$

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These three boundary conditions are necessary to permit the determination of both $\tilde{p}_N(z)$ and k_N.

Equation (2) is integrated piecewise inside the internal domain of the combustion chamber (Fig. 1), which is separated into several adjacent subdomains located between points of discontinuity of dR/dz [in this case, $R(z)$ is assumed continuous]. At each subdomain interface, the continuity of $\tilde{p}_N(z)$ and $d\tilde{p}_N/dz(z)$ is imposed. With the rigid-wall assumption considered here, Eq. (1) can be rewritten as

$$k^2 = k_N^2 + \frac{ik_N}{E_N^2}\left[-\int_{S_b} \tilde{p}_N^2 (A_b^* + M_b)\mathrm{d}s + \int_{S_L} \tilde{p}_N^2 (A_L + \bar{M}_L)\mathrm{d}s\right] \tag{4}$$

A_b^* is the burning surface acoustic admittance corrected from the effects of the developing acoustic boundary layer above it. Since it is assumed that $A_b = 0$, A_b^* is the acoustic boundary-layer admittance computed according to Ref. 5 or 6. The aft closure plane acoustic admittance A_L is classically determined by integrating the linearized (with respect to $\epsilon = |\tilde{p}|/\bar{p}_0$) one-dimensional unsteady equation of motions inside the nozzle. The integration method is the fourth-order Runge-Kutta method initiated at the sonic throat section and carried in the upstream direction up to the aft closure plane at $z = L$. This determination of A_L, which is, of course, limited to the case of axial acoustic modes, has proved to be very reliable and to yield results identical to those presented in Ref. 7, which were obtained either experimentally or theoretically.

Taking the real and imaginary part of Eq. (4) yields the formulas for the perturbed frequency f and for the damping coefficient α. For stability analyses, α is of primary importance and its expression obtained from Eq. (4) can be further rearranged as a sum of several contributions respectively related to burning surface admittance corrected for viscous effects, convection of acoustic energy at the burning surface, aft closure plane acoustic admittance, and convection of acoustic energy at the aft closure plane,

$$\alpha = \alpha A_b^* + \alpha \bar{M}_b + \alpha A_L + \alpha \bar{M}_L \tag{5}$$

Since k_N is purely real, there is a one-to-one correspondence between the right-hand side of Eq. (5) and the real part of the term of Eq. (4) between brackets.

In order to study the sensitivity of the stability analyses to the position of the aft closure plane, stability analyses have been performed for the four positions of the aft closure plane as shown on Fig. 2.

Table 1 summarizes the results that are obtained for two different values of the web distance burned: $e_b = 0$ and 3 mm [$(e_b)_{\text{burnout}} = 5$ mm]. From Table 1, it is clear that the predictions for both f and α are very sensitive to the location of the aft closure plane. Indeed, for initial conditions ($e_b = 0$ mm), the computed values of f and α are found to be 5636–6366 Hz and 895–405 s^{-1}, respectively, only because of the differing locations of the aft closure plane. It is worthwhile to note that the major source of discrepancy from α comes from the evaluation of the nozzle losses α_{A_L} and $\alpha_{\bar{M}_L}$ and can be traced back to the computed value of $\tilde{p}_N$ at the aft closure plane ($z = L$). For instance, this can be illustrated by considering $\alpha_{\bar{M}_L}$, which is proportional to $\bar{M}_L S_L \tilde{p}_N^2(L)$. For an incompressible flowfield without mass injection, $\bar{M}_L S_L = \text{const}$; thus, $\bar{M}_L/S_L$ does not depend on the precise location of the aft closure plane. However, $\tilde{p}_N(L)$ does depend on this location where the rigid-wall boundary condition, $(d\tilde{p}_N/dz)_L = 0$, is imposed. Consequently, it appears that the acoustic mode determination plays an important role, particularly when evaluating the nozzle losses.

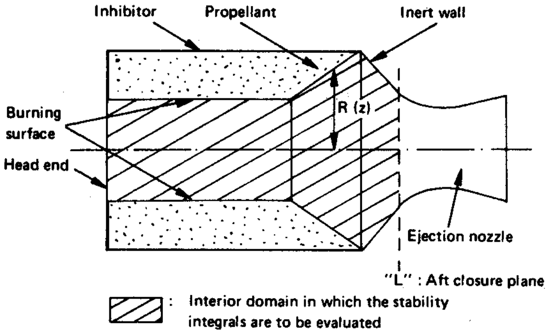
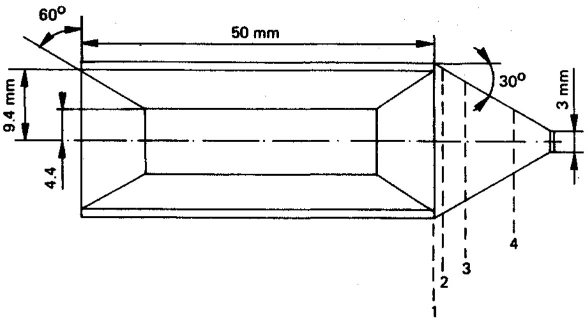


Fig. 1 Typical rocket motor geometry.



Aft closure plane	Distance from head end L (mm)	Radius at closure plane R_L (mm)	Mach number $\bar{M}_L$
1	50	9.4	0.015
2	51.6	8.5	0.018
3	54.3	6.9	0.028
4	61.1	3	0.150

Fig. 2 Motor geometry for the test case.

Table 1 Classical computations: $p = 30b$, $a_0 = 1056$ m/s, $\gamma = 1.213$, $A_b = 0$

e_b, mm	Aft closure plane no.	f_N, Hz	f, Hz	$\alpha_{A_b^*}$, s^{-1}	α_{M_b}, s^{-1}	α_{A_L}, s^{-1}	α_{M_L}, s^{-1}	α_{TOT}, s^{-1}
0	1	7517	5636	440	−425	173	707	895
	2	7108	6073	395	−382	113	561	687
	3	6710	6310	362	−351	69	431	510
	4	6396	6366	347	−338	39	356	405
3	1	9210	7973	155	−146	122	373	505
	2	8922	8115	144	−137	91	344	443
	3	8584	8212	134	−127	62	315	383
	4	8264	8235	127	−121	35	312	353

Table 2 Modified computations: $p = 30b$, $a_0 = 1056$ m/s, $\gamma = 1.213$, $A_b = 0$

e_b, mm	Aft closure plane No.	f_N, Hz	f, Hz	$\alpha_{A_b^*}$, s^{-1}	α_{Mb}, s^{-1}	α_{AL}, s^{-1}	α_{ML}, s^{-1}	α_{TOT}, s^{-1}
0	1	6376	6364	441	−429	82	403	496
	2	6376	6365	411	−400	70	384	465
	3	6358	6348	378	−368	56	361	427
	4	6375	6366	349	−340	39	357	405
3	1	8241	8236	154	−147	88	319	415
	2	8241	8236	146	−139	75	314	396
	3	8221	8216	136	−129	58	309	375
	4	8239	8235	127	−121	36	318	360

Modified Stability Analysis

One way to reduce the observed sensitivity of the stability calculations to the position of the aft closure plane is to incorporate into the acoustic computations information on the location of the closure plane relative to the nozzle sonic throat. This can be achieved through the nozzle admittance A_L, and the rigid-wall condition applied at the aft closure plane can be replaced by the following simple admittance condition:

$$\left(\frac{\mathrm{d}\tilde{p}_N}{\mathrm{d}z}\right)_{z=L} = \gamma k_N \bar{M}_L \mathrm{Im}[A_L(k_N)]\tilde{p}_N \tag{6}$$

For the sake of simplicity, only the imaginary part of A_L is fed back to the reference acoustic mode. This can be justified by Fig. 3, which shows the variations of both the real and imaginary part of A_L vs a dimensionless nozzle frequency. Indeed, Fig. 3 shows that Im(A_L) is greater than Re(A_L) for oscillations whose wavelength is larger than the nozzle's own length (roughly $\Omega < 1$ on Fig. 3). The major advantage of considering only Im(A_L) is that both $\tilde{p}_N(z)$ and k_N remain purely real, which greatly simplifies the stability analysis. The penalty for this improved acoustic boundary condition is the necessity to iterate between the acoustic mode computation and the nozzle admittance computation since k_N and $\tilde{p}_N(z)$ are now functions of A_L. One way to accomplish these iterations is to start with $A_L = 0$ in Eq. (6), find k_N, then compute $A_L(k_N)$, which in turn will lead to a new acoustic mode. This iterative procedure typically converges in less than five iterations. Moreover, Eq. (4) has to be modified to take into account the new boundary conditions for the reference acoustic mode and now reads

$$k^2 = k_N^2 + \frac{ik_N}{E_N^2}\left\{-\int_{S_b} \tilde{p}_N^2(A_b^* + \bar{M}_b)\mathrm{d}s + \int_{S_L} \tilde{P}_N^2[Re(A_L) + \bar{M}_L]\mathrm{d}s\right\} \tag{7}$$

Then the preceding stability calculations were performed again with the new acoustic mode determination. The results are summarized in Table 2 and show a greatly improved stability prediction accuracy. Comparisons between Tables 1 and 2 show that the classical stability calculations assuming rigid walls everywhere and closing the internal domain at the entrance to the converging nozzle (position 1) lead to overpredicted nozzle losses. Moreover, it is interesting to note that position 4 of the aft closure plane yields results independent of the acoustic boundary conditions. This shows that rigid-wall assumptions could be used safely, provided a proper location of the aft closure plane can be selected. However, since such a selection can turn out to be difficult, it seems preferable to use the modified approach. It is also interesting to note that the results for $e_b = 3$ mm seem to be less sensitive to the acoustic mode determination. This could be due to the internal geometry being closer to a pure cylinder (exact cylinder for $e_b = 5$ mm).

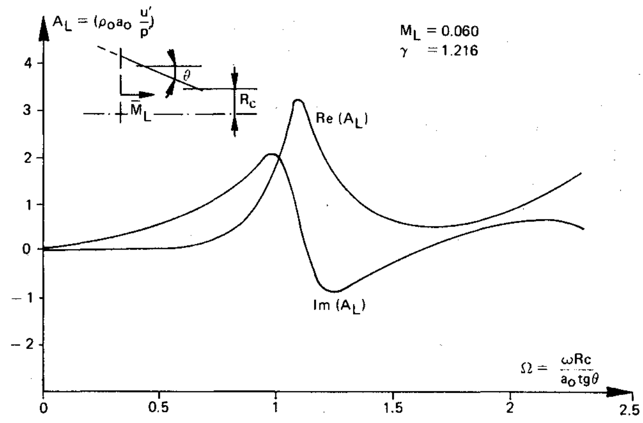


Fig. 3 Typical nozzle acoustic admittance curves.

Application of the proposed modified stability analysis to two- or three-dimensional analyses seems an easy way of improving existing rocket engine stability models. However such an extension would require multidimensional nozzle admittance data that fully incorporate multidimensional effects (such as two-dimensional mean flow and nozzle admittance variations across the nozzle entrance plane). Such data were first reported theoretically in Ref. 8 and seem adequately to complement the pioneering results of Ref. 7. However, such works are in an early stage of development, and the effects of rotational mean flow remain to be investigated. Work still needs to be done before accurate multidimensional stability analyses can be performed in complex configurations.

Conclusions

Classical stability analyses relying on reference acoustic modes determined through rigid-wall boundary conditions have been shown to be inaccurate. It is inferred that these discrepancies are all the more noticeable because the chamber length cannot be assumed large compared to the nozzle length and also because the internal geometry is complex. A simple admittance condition to be imposed at the aft closure plane of the internal cavity is proposed that significantly improves the accuracy of the stability predictions for such complex internal geometries. In particular, it has been shown that classical stability analyses tend to overpredict the rocket motor stability boundary through overpredicted nozzle losses. However, application of the improved procedure to two- or three-dimensional analyses is presently precluded due to the lack of two- or three-dimensional nozzle admittance data. Availability of such data should lead to improved accuracy of rocket engine stability predictions.

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